CORREZIONE TEMA 12.7.05

ESERCIZIO 1

a) $\mathbb{E}(X^2) = Var(X) + [\mathbb{E}(X)]^2 = 1 + \mu^2 = \tau(\mu)$

$$\ln f = \ln \frac{1}{\sqrt{2\pi}} - \frac{1}{2}(x_i - \mu)^2$$

$$\ln L = n \ln \frac{1}{\sqrt{2\pi}} - \frac{1}{2} \Sigma (x_i - \mu)^2$$

$$\frac{\partial \ln L}{\partial \mu} = -\frac{1}{2} \cdot 2 \Sigma (x_i - \mu) \cdot (-1)$$

$$= \Sigma (x_i - \mu) = 0 \quad \Rightarrow \Sigma x_i - n\mu = 0$$

$$\Rightarrow \hat{\mu}_{ML} = \frac{\Sigma x_i}{n} = \overline{X}$$

Per il principio di invarianza:

$$T = \tau(\hat{\mu}_{ML}) = 1 + \overline{X}^2$$

b) $$\mathbb{E}(T) = 1 + \mathbb{E}(\overline{X}^2) = 1 + \{Var(\overline{X}) + [\mathbb{E}(\overline{X})]^2\}$$

$$= 1 + \left\{\frac{Var(X)}{n} + \mu^2\right\} = (1 + \mu^2) + \frac{1}{n}$$

$$= \tau(\mu) + \frac{1}{n}$$

$T' = T - \frac{1}{n} = 1 + \overline{X}^2 - \frac{1}{n}$ è corretto per $\mathbb{E}(X^2)$

c) $$\Sigma \frac{\partial}{\partial \mu} \ln f(x_i ; \mu) = \Sigma (x_i - \mu) = \Sigma x_i - n\mu$$

$$= n \left\{ \frac{\Sigma x_i}{n} - \mu \right\}$$

$\tau^*(\mu)$

$\Rightarrow$ La varianza di T' non può raggiungere il limite inferiore di Rao-Cramèr perché ...

ESERCIZIO 2.

a) H_0: Quotidiano e Ceto INDIPENDENTI ($\alpha = 0{,}05$)

Tabella frequenze teoriche ($\hat{n}_{ij}$):

	A	B	C	TOT
Pov	38	38	38	114
M-B	25	25	25	75
M-A	53	53	53	159
B	4	4	4	12
TOT	120	120	120	360

Calcolo $X^2 = \sum_i \sum_j \frac{(n_{ij} - \hat{n}_{ij})^2}{\hat{n}_{ij}}$:

	A	B	C	TOT
Pov	4,4474	1,2895	0,9474	6,6842
M-B	1,96	0,04	1,44	3,44
M-A	0,3019	0,6792	0,0755	1,0566
B	1	0	1	2
TOT	7,7093	2,0087	3,4628	13,1808

$13{,}1808 = X^2$

Valore critico: $\chi^2_{(6;\,0{,}90)} = 10{,}6$

$\Rightarrow$ rifiuto H_0: c'è dipendenza fra quotidiano letto e ceto sociale del lettore

b) $\hat{p}_{POV} = \frac{51}{114} = 0{,}4474$ $\quad$ $\hat{q}_{POV} = 0{,}5526$

$\hat{p}_{M\text{-}A} = \frac{49}{159} = 0{,}3082$ $\quad$ $\hat{q}_{M\text{-}A} = 0{,}6918$

$$I(p_{POV} - p_{M\text{-}A})$$

$$= \left\{ (0{,}4474 - 0{,}3082) \pm z_{(0{,}985)} \cdot \sqrt{\frac{0{,}4474 \cdot 0{,}3082}{114} + \frac{0{,}3082 \cdot 0{,}6918}{159}} \right.$$

$$= \left\{ 0{,}1392 \pm \underbrace{2{,}17 \cdot 0{,}0592}_{0{,}1286} \right\}$$

$$= \left\{ 0{,}0106 \; ; \; 0{,}2678 \right\}$$

ESERCIZIO 3.

STIMA di β_0, β_1 :

$$Cov(X,Y) = \frac{1}{4} \sum x_i \cdot y_i - \bar{x} \cdot \bar{y}$$

$$= \frac{1}{4} 9584 - 45{,}5 \cdot 52 = 30$$

$$Var(X) = \frac{1}{4} \sum x_i^2 - \bar{x}^2 = \frac{1}{4} 8406 - 45{,}5^2 = 31{,}25$$

$$Var(Y) = \frac{1}{4} \sum y_i^2 - \bar{y}^2 = \frac{1}{4} \cdot 10990 - 52^2 = 43{,}5$$

$$\rightarrow \hat{\beta}_1 = \frac{30}{31{,}25} = 0{,}96$$

$$\rightarrow \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x} = 52 - 0{,}96 \cdot 45{,}5 = 8{,}32$$

a) Per l'intervallo di confidenza su β_0 occorre la stima di σ^2.

$$DEV.\ RES = Dev(Y) - \hat{\beta}_1^2 \cdot Dev(X)$$

$$= 4 \cdot 43{,}5 - 0{,}96^2 \cdot [4 \cdot 31{,}25]$$

$$= 174 - 0{,}96^2 \cdot 125 = 58{,}8$$

$$\tilde{\sigma}^2 = \frac{1}{n-2} \cdot DEV.RES. = \frac{1}{2} \cdot 58{,}8 = 29{,}4$$

$$\hat{Var}(\hat{\beta}_0) = \frac{\tilde{\sigma}^2}{n} \left[1 + \frac{\bar{x}^2}{Var\, X} \right] = \frac{29{,}4}{4} \left[1 + \frac{45{,}5^2}{31{,}25} \right]$$

$$= 494{,}2728$$

Valore critico al 90% :

$$t_{(2\ gdl;\ 0{,}95)} = 2{,}920$$

$$\rightarrow I(\beta_0) = \hat{\beta}_0 \pm t_{(2;0,95)} \cdot \sqrt{\hat{Var}(\hat{\beta}_0)}$$

$$= 8,32 \pm 2,920 \cdot \sqrt{494,2728}$$

$$= \{-56,5982 ; 73,2381\}$$

b) $\begin{cases} H_0: & \beta_1 = 1 \\ H_1: & \beta_1 < 1 \end{cases} \qquad \alpha = 0,10$

$$\hat{Var}(\hat{\beta}_1) = \frac{\tilde{\sigma}^2}{n} \cdot \frac{1}{Var\,X} = \frac{29,4}{4} \cdot \frac{1}{31,25} = 0,2352$$

$$T = \frac{\hat{\beta}_1 - \beta_1(H_0)}{\sqrt{\hat{Var}(\hat{\beta}_1)}} = \frac{0,96 - 1}{\sqrt{0,2352}} = -0,0825$$

Regione critica: $C = \{T < -t_{(2;\,0,90)}\} = \{T < -1,886\}$

$\Rightarrow$ accetto $H_0: \beta_1 = 1 \quad (\alpha = 0,10)$

c) $\hat{\mu}(x = 35) = 8,32 + 0,96 \cdot 35 = 41,92$

$$\hat{Var}(\hat{\mu}[x=35]) = \frac{\tilde{\sigma}^2}{n}\left[1 + \frac{(35-\bar{x})^2}{Var\,X}\right]$$

$$= \frac{29,4}{4}\left[1 + \frac{(35-45,5)^2}{31,25}\right] = 33,2808$$

$$I(\mu[35]) = \hat{\mu}[35] \pm t_{(2;\,0,90)} \cdot \sqrt{\hat{Var}(\hat{\mu}[35])}$$

$$= 41,92 \pm 1,886 \cdot 5,7690$$

$$= \{31,0398 ; 52,8002\}$$