CORREZIONE TEMA 14.06.05

ESERCIZIO 1.

a) $\ln f = \ln \frac{1}{\sqrt{2\pi}} - \frac{1}{2} \ln \sigma^2 - \frac{1}{2\sigma^2}(x_i-\mu)^2$

$\ln L = n \ln \frac{1}{\sqrt{2\pi}} - \frac{n}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} \sum_i (x_i-\mu)^2$

$\frac{\partial \ln L}{\partial \sigma^2} = -\frac{n}{2\sigma^4} + \frac{1}{2\sigma^4} \sum (x_i-\mu)^2 = 0$

$\Rightarrow \quad \tilde{\sigma}^2_{ML} = \frac{1}{n} \sum (x_i-\mu)^2 = \frac{1}{n} \sum (x_i-4)^2 = 1,3943$

b) $\tilde{Q}_3 = x(0,75)$

$= 4 + \underbrace{\tilde{\sigma}_{ML}}_{1,1808} \cdot \underbrace{z(0,75)}_{0,674}$ (TAVOLE)

$= 4,7964$

c) $I(\sigma) = \left\{ \sqrt{\frac{\sum (x_i-4)^2}{\chi^2_{(7;\,0,975)}}} \; ; \; \sqrt{\frac{\sum (x_i-4)^2}{\chi^2_{(7;\,0,025)}}} \right\}$

$= \left\{ \sqrt{\frac{9,76}{16}} \; ; \; \sqrt{\frac{9,76}{1,69}} \right\}$

$= \left\{ 0,7810 \; ; \; 2,4032 \right\}$

$I\left(\frac{\sigma}{\mu}\right) = \left\{ \frac{0,7810}{4} ; \frac{2,4032}{4} \right\} = \left\{ 0,1953 ; 0,6008 \right\}$

ESERCIZIO 2.

a)

GRUPPO	n_J	$\bar{X}_J$	Dev_J	S^2_J
Nord	22	50	13'125	625
Centro	25	84,5	11'025	459,375
Sud	16	82,0312	4052,734	270,182
	63	$\bar{X}=$ 71,8254	28202,734	

$$V = \frac{S^2_N}{S^2_C} = \frac{625}{459,375} = 1,3605 \quad > 1$$

Basta cfr con $F_{(21,24;\,0,99)} = ?$

cons. $F_{(20,20;\,0,99)} = 2,94$

$F_{(20,30;\,0,99)} = 2,55$

accetto Ho : le varianze sono

b) $DF = (50-71,8254)^2 \cdot 22 + (84,5-71,8254)^2 \cdot 25 + (82,0312-71,8254)^2 \cdot 16 = 16'162,34$

$DN = \Sigma\, Dev_J = 28'202,734$

$$V = \frac{DF/(K-1)}{DN/(n-K)} = \frac{16'162,34/2}{28'202,734/60} = 17,1923$$

$F_{critico} = F_{(2,60;\,0,99)} = 4,98$

→ Rifiuto Ho : le medie sono ≠

c) $\begin{cases} H_0: \mu_S = 75 \\ H_1: \mu_S > 75 \end{cases}$ $\qquad X_S \sim N(\mu_S, \sigma_S^2 = 250 \text{ } \underline{\text{NOTA}})$

$$C = \left\{ \bar{X} > \mu_S(H_0) + z(1-\alpha) \cdot \frac{\sigma_S}{\sqrt{n_S}} \right\}$$

$$= \left\{ \bar{X} > 75 + \underbrace{z(0{,}975)}_{1{,}96} \cdot \sqrt{\frac{250}{16}} \right\}$$

$$= \left\{ \bar{X} > 82{,}7476 \right\}$$

Essendo $\bar{x}_S = 82{,}0312 \notin C \Rightarrow$ accetto H_0, cioè ritengo $\mu_S = 75$ $\quad (\alpha = 0{,}025)$

d) • $\beta = \text{Prob}(\text{acc. } H_0 \mid H_1)$

$$= \text{Prob}(\bar{X} < 82{,}7476 \mid \mu_1 = 80)$$

$$= \Phi\left(\frac{82{,}7476 - 80}{\sqrt{250/16}}\right) = \Phi(0{,}6951)$$

$$\cong \Phi(0{,}70) = 0{,}7580$$

$$\pi(\mu_1) = 1 - \beta = 0{,}2420$$

• $\beta = \text{Prob}(\bar{X} < 82{,}7476 \mid \mu_2 = 85)$

$$= \Phi\left(\frac{82{,}7476 - 85}{\sqrt{250/16}}\right) = \Phi(-0{,}57)$$

$$= 1 - \Phi(0{,}57) = 1 - 0{,}7157 = 0{,}2843 \ \downarrow$$

$$\pi(\mu_2) = 0{,}7157 \ \uparrow$$