CORREZIONE TEMA 19.2.05

ESERCIZIO 1.

a. $\ln p(x_i;\lambda) = -\lambda + x_i \ln\lambda - \ln x!$

$$\log \mathcal{L} = \sum_i \ln p(x_i;\lambda) = -n\lambda + \ln\lambda \sum_1^n x_i - \sum_i \ln x_i!$$

$$\frac{\partial \log \mathcal{L}}{\partial\lambda} = -n + \frac{1}{\lambda}\sum x_i = 0 \longrightarrow \hat{\lambda}_{ML} = \overline{X} = T_1$$

$$\frac{\partial^2 \log \mathcal{L}}{\partial\lambda^2} = -\frac{1}{\lambda^2}\sum x_i < 0$$ massimo

CORRETTEZZA:

$$\mathbb{E}(\overline{X}) = \mathbb{E}(X) = \lambda$$

CONSISTENZA QUADRATICA:

$$EQM(\overline{X}) = Var(\overline{X}) = \frac{Var(X)}{n} = \frac{\lambda}{n} \xrightarrow[n\to\infty]{} 0$$

b. $$L = \frac{1}{n\,\mathbb{E}\left(-\frac{\partial^2 \ln f}{\partial\lambda^2}\right)} = \frac{1}{n\,\mathbb{E}\left(-\left[-\frac{X}{\lambda^2}\right]\right)}$$

$$= \frac{1}{\frac{n}{\lambda^2}\cdot\mathbb{E}(X)} = \frac{1}{\frac{n}{\lambda^2}\cdot\cancel{\lambda}} = \underbrace{\frac{\lambda}{n} = EQM(T_1)}$$

quindi $T_1 = \overline{X}$ è stimatore non distorto a varianza uniformem. minima per λ

c. $$\tau(\lambda) = \text{Prob}(X=0) = e^{-\lambda}$$

Per il principio di invarianza:

$$T_2 = \tau(\hat{\lambda}_{ML}) = e^{-\overline{X}}$$

ESERCIZIO 2.

a) Ho: indipendenza

Frequenze teoriche: $\hat{n}_{ij}$

GENITORE \ STUDENTE	D	C	I	TOT
D	394,1965	195,3672	326,4363	916
C	213,0211	105,5751	176,4039	495
I	189,7824	94,0578	157,1598	441
TOT	797	395	660	1852

Contingenze: $n_{ij} - \hat{n}_{ij}$

▸ Rapporti : $(n_{ij} - \hat{n}_{ij})^2 / \hat{n}_{ij}$

GENITORE \ STUDENTE	D	C	I	TOT
D	2,2533	4,1189	0,0063	6,3785
C	1,8817	2,5553	0,0733	4,5103
I	0,5042	1,5163	0,0297	2,0502
TOT	4,6392	8,1905	0,1093	12,9390 $= X^2$

valore critico:

$$\chi^2_{(r-1)\cdot(c-1);\,0,95} = \chi^2_{4;\,0,95} = 9,4877$$

→ essendo $X^2 > 9,4877$ rifiuto Ho:
le preferenze politiche degli studenti non sono indipendenti da quelle dei genitori

b) $\hat{p}_S = \frac{797}{1852} = 0{,}4303$

$\hat{p}_G = \frac{916}{1852} = 0{,}4946$

I.C: $\hat{p}_S - \hat{p}_G \pm z_{(1-\frac{\alpha}{2})} \sqrt{\frac{\hat{p}_S \hat{q}_S}{n_S} + \frac{\hat{p}_G \hat{q}_G}{n_G}}$

$= -0{,}0643 \pm z_{(0{,}99)} \cdot \sqrt{\frac{0{,}2451}{1852} + \frac{0{,}24997}{1852}}$

$= -0{,}0643 \pm \underbrace{2{,}326 \cdot 0{,}0164}_{0{,}0380} = \{-0{,}1023 ; -0{,}0263\}$

ESERCIZIO 3

$\bar{X} = 13$ $\qquad$ $\bar{Y} = 139{,}625$

$Var(X) = 9{,}5$ $\qquad$ $Var(Y) = 2005{,}4844$

$Cov(X,Y) = 105{,}625$

$\rightarrow \hat{\beta}_1 = \frac{105{,}625}{9{,}5} = 11{,}1184$

$\rightarrow \hat{\beta}_0 = 139{,}625 - 11{,}1184 \cdot 13 = -4{,}9145$

a) $DR = Dev(Y) - \hat{\beta}_1^2 \cdot Dev(X) =$

$= 16'043{,}875 - 11{,}1184^2 \cdot 76 = 6648{,}8092$

$\tilde{\sigma}^2 = \frac{DR}{n-2} = \frac{6648{,}8092}{6} = 1108{,}1349$

I.C. $= \left\{ \frac{DR}{\chi^2_{(6;0{,}99)}} ; \frac{DR}{\chi^2_{(6;0{,}01)}} \right\}$

$= \left\{ \frac{6648{,}8092}{16{,}8119} , \frac{6648{,}8092}{0{,}8721} \right\} = \{395{,}483 ; 7624{,}053\}$

d) $\begin{cases} H_0: \beta_1 = 11 \\ H_1: \beta_1 > 11 \end{cases}$

$$T = \frac{\hat{\beta}_1 - 11}{\sqrt{\frac{\tilde{\sigma}^2}{n} \cdot \frac{1}{Var(X)}}} = \frac{0,1184}{\sqrt{\frac{1108,1394}{76}}} = \frac{0,1184}{3,8185} = 0,0310$$

$t_{(6\,;\,0,99)} = 3,143$ (valore critico)

→ Accetto H_0

e) $\hat{\mu}(x = 12) = -4,9145 + 11,1184 \cdot 12 = 128,5066$

$$\text{i.c}: \hat{\mu} \pm t_{(6\,;\,0,975)} \cdot \sqrt{\frac{\tilde{\sigma}^2}{n} \cdot \left[1 + \frac{(12 - \bar{X})^2}{Var(X)} \right]}$$

$$= 128,5066 \pm 2,447 \cdot \sqrt{\frac{1108,1349}{8} \cdot \left[1 + \frac{(12-13)^2}{9,5} \right]}$$

$$= 128,5066 \pm \underbrace{2,447 \cdot 12,3733}_{30,2774}$$

$$= \{ 98,2292 \,;\, 158,7840 \}$$