CORREZIONE TEMA 21.1.05 STAT IIB

ESERCIZIO 1.

a) $\ln f(x_i;\gamma) = -\ln\gamma - \frac{1}{\gamma} x_i$

$\ln L = -n\ln\gamma - \frac{1}{\gamma}\sum_1^n x_i$

$\frac{\partial \ln L}{\partial\gamma} = -\frac{n}{\gamma} + \frac{1}{\gamma^2}\sum x_i = 0 \rightarrow \hat{\gamma} = \bar{X} = T$

b) Essendo $X \sim Esp\left(\theta = \frac{1}{\gamma}\right)$ si ha: $E(X) = \gamma$

$Var(X) = \gamma^2$

CORRETTEZZA di T:

$E(T) = E(\bar{X}) = E(X) = \gamma \longrightarrow$ corretto per γ

CONSISTENZA QUADRATICA di T:

$EQM(T) = Var(T) = Var(\bar{X}) = \frac{Var(X)}{n} = \frac{\gamma^2}{n} \xrightarrow[n\to\infty]{} 0$

quindi T è q-consistente per γ

c) $L = \frac{1}{n\,E\left(\frac{\partial \ln f}{\partial\gamma}\right)^2} = \frac{1}{n\,E\left(-\frac{1}{\gamma}+\frac{1}{\gamma^2}\cdot X\right)^2} = \frac{1}{\frac{n}{\gamma^4}\cdot \underbrace{E(X-\gamma)^2}_{Var(X)=\gamma^2}}$

$= \frac{1}{\frac{n}{\gamma^{\cancel{4}\,2}}\cdot\cancel{\gamma^2}} = \frac{\gamma^2}{n} = EQM(T)$

$\rightarrow$ T è l'unico stimatore a varianza uniformemente minima per il parametro γ

ESERCIZIO 2.

a) Stima di λ:

$$\hat{\lambda} = \bar{x} = \frac{\sum n_j \cdot x_j}{n} = \frac{300}{100} = 3$$

x_j	π_j	$\hat{n}_j$		n_j	$\frac{(n_j - \hat{n}_j)^2}{\hat{n}_j}$
0	0,0498	4,98	19,91	20	0,0004
1	0,1494	14,94			
2	0,2240	22,40		21	0,0880
3	0,2240	22,40		24	0,1137
4	0,1680	16,80		17	0,0023
5	0,1008	10,08		11	0,0836
≥6	0,0839	8,39		7	0,2308
	1	100			0,5188 = X^2

$$\left(\pi_j = \frac{e^{-3} \cdot 3^{x_j}}{x_j!} \qquad j = 0,1,2 \dots \right)$$

Valore critico: $\chi^2_{(k-1-s;\,0,95)} = \chi^2_{(4;\,0,95)} = 9,49$

Essendo $X^2 < 9,49$ $\rightarrow$ accetto Ho: i dati provengono da una distr. Poisson

b) $$\hat{p} = \frac{20+21+24}{100} = \frac{65}{100} = 0,65$$

$$\hat{p} \pm z_{(1-\frac{\alpha}{2})} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0,65 \pm z_{(0,99)} \sqrt{\frac{0,65 \cdot 0,35}{100}}$$

$$= 0,65 \pm 2,326 \cdot 0,0477$$

$$= 0,65 \pm 0,1109 \; \begin{cases} 0,5391 \\ 0,7609 \end{cases}$$

ESERCIZIO 3.

$$r^2 = \frac{\hat{\beta}_1^2 \cdot Dev(X)}{Dev(Y)}$$

$$\rightarrow \quad Dev(Y) = \frac{\hat{\beta}_1^2 \cdot Dev(X)}{r^2} = \frac{1{,}0353^2 \cdot 363{,}9422}{0{,}7197}$$

$$= 542{,}0175$$

$$\rightarrow \quad DR = (1 - r^2) \cdot Dev(Y) = 151{,}9275$$

$$\tilde{\sigma}^2 = \frac{DR}{n-2} = \frac{151{,}9275}{7} = 21{,}7039$$

a) $$\hat{\beta}_1 \pm t_{(n-2;\, 1-\alpha/2)} \cdot \sqrt{\frac{\tilde{\sigma}^2}{n} \cdot \frac{1}{Var(X)}}$$

$$= 1{,}0353 \pm \underbrace{t_{(7;\, 0{,}99)}}_{2{,}9998} \cdot 0{,}2442$$

$$= \{ 0{,}3027 \; ; \; 1{,}7679 \}$$

b) $$T = \frac{\hat{\beta}_0}{\sqrt{\frac{\tilde{\sigma}^2}{n}\left(1 + \frac{\bar{x}^2}{Var(X)}\right)}} = \frac{-0{,}3893}{2{,}7806} = -0{,}1400$$

$$C = \{ T < -\underbrace{t_{(n-2;\, 1-\alpha)}}_{t_{(7;\, 0{,}99)}} \} = \{ T < -2{,}9998 \}$$

Essendo $-0{,}1400 \notin C \rightarrow$ acc. H_0

c) I.C. per $\tilde{\sigma}^2$ $$= \left\{ \frac{DR}{\chi^2_{(n-2;\, 1-\alpha/2)}} \; ; \; \frac{DR}{\chi^2_{(n-2;\, \alpha/2)}} \right\}$$

$$= \left\{ \frac{151{,}9275}{14{,}067} \; ; \; \frac{151{,}9275}{2{,}167} \right\}$$

$$= \{ 10{,}8002 \; ; \; 70{,}0983 \}$$

IC. per σ $= \{ \sqrt{10{,}8002} \; ; \; \sqrt{70{,}0983} \} = \{ 3{,}2864 ; \; 8{,}3725 \}$