$$\int \frac{1}{x^3+1}\,dx =$$

$$= \int \frac{1}{(x+1)(x^2-x+1)}\,dx =$$

$$= \int \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-x+1)}\,dx =$$

$$= \int \frac{A(x^2-x+1)+(Bx+C)(x+1)}{(x+1)(x^2-x+1)}\,dx =$$

$$= \int \frac{Ax^2-Ax+A+Bx^2+Bx+Cx+C}{(x+1)(x^2-x+1)}\,dx =$$

$$= \int \frac{x^2(A+B)+x(B+C-A)+(A+C)}{(x+1)(x^2-x+1)}\,dx$$

$$\begin{cases} A+B=0 \\ B+C-A=0 \\ A+C=1 \end{cases} \quad \begin{cases} A=-B \\ B+C+B=0 \\ -B+C=1 \end{cases} \quad \begin{cases} \sim \\ C=-2B \\ -B-2B=1 \end{cases} \quad \begin{cases} \sim \\ \sim \\ B=-\frac{1}{3} \end{cases}$$

$$\begin{cases} A=-B \\ C=-2\left(-\frac{1}{3}\right) \\ B=-\frac{1}{3} \end{cases} \quad \begin{cases} A=\frac{1}{3} \\ C=\frac{2}{3} \\ B=-\frac{1}{3} \end{cases}$$

$$= \int \frac{\frac{1}{3}}{(x+1)} + \frac{-\frac{1}{3}x+\frac{2}{3}}{(x^2-x+1)}\,dx = \frac{1}{3}\ln|x+1| - \frac{1}{3}\int \frac{x-2}{x^2-x+1} =$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{3}\cdot\frac{1}{2}\int \frac{2x-4}{x^2-x+1}\,dx =$$

SEGUE →

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$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\int \frac{2x-1-3}{x^2-x+1}\,dx =$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\int \frac{2x-1}{x^2-x+1} - \frac{3}{x^2-x+1}\,dx =$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln|x^2-x+1| + \frac{3}{6}\int \frac{1}{x^2-x+1}\,dx =$$

$\Delta < 0$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2-x+1) + \frac{1}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\frac{1}{4}+1}\,dx =$$

$$= \text{''} - \text{''} + \frac{1}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\,dx =$$

$$= \text{''} - \text{''} + \frac{1}{2}\int \frac{1}{\frac{3}{4}\left(\frac{4}{3}\left(x-\frac{1}{2}\right)^2+1\right)}\,dx =$$

$$= \text{''} - \text{''} + \frac{1}{2}\cdot\frac{4}{3}\int \frac{1}{\left[\frac{2}{\sqrt{3}}\left(x-\frac{1}{2}\right)\right]^2+1}\,dx =$$

$$= \text{''} - \text{''} + \frac{2}{3}\int \frac{1}{\left(\frac{2}{\sqrt{3}}x-\frac{1}{\sqrt{3}}\right)^2+1}\,dx =$$

SOSTITUZ.: $\frac{2}{\sqrt{3}}x - \frac{1}{\sqrt{3}} = z \rightarrow x = \left(z + \frac{1}{\sqrt{3}}\right)\frac{\sqrt{3}}{2} \rightarrow \frac{dx}{dz} = \frac{\sqrt{3}}{2}$

$dx = \frac{\sqrt{3}}{2}\,dz$

$$= \text{''} - \text{''} + \frac{2\sqrt{3}}{3\cdot 2}\operatorname{arctg} z + k =$$

$$= \boxed{\frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2-x+1) + \frac{\sqrt{3}}{3}\operatorname{arctg}\left(\frac{2}{\sqrt{3}}x - \frac{1}{\sqrt{3}}\right) + k}$$