

$$\int \frac{1}{\operatorname{sen} x} dx =$$

INT-S1-032

$$\left(\text{FORMULA TRIGONOMETRICA} \quad \operatorname{sen} x = \frac{2 \operatorname{tg} \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} \right)$$

$$= \int \frac{1 + \operatorname{tg}^2 \frac{x}{2}}{2 \operatorname{tg} \frac{x}{2}} dx =$$

$$= \int \frac{1}{2 \operatorname{tg} \frac{x}{2}} + \frac{\operatorname{tg}^2 \frac{x}{2}}{2 \operatorname{tg} \frac{x}{2}} dx =$$

$$= \frac{1}{2} \int \frac{1}{\operatorname{tg} \frac{x}{2}} dx + \frac{1}{2} \int \operatorname{tg} \frac{x}{2} dx =$$

$$= \frac{1}{2} \int \operatorname{cotg} \frac{x}{2} dx + \frac{1}{2} \int \operatorname{tg} \frac{x}{2} dx =$$

$$= \frac{1}{2} \int \frac{\cos \frac{x}{2}}{\operatorname{sen} \frac{x}{2}} dx + \frac{1}{2} \int \frac{\operatorname{sen} \frac{x}{2}}{\cos \frac{x}{2}} dx$$

$$\text{SOSTITUZIONE} \quad \frac{x}{2} = z$$

$$x = 2z$$

$$z' = \frac{dx}{dz} = 2$$

$$= \frac{1}{2} \int \frac{\cos z}{\operatorname{sen} z} 2 dz + \frac{1}{2} \int \frac{\operatorname{sen} z}{\cos z} 2 dz = \int \frac{\cos z}{\operatorname{sen} z} dz + \int \frac{\operatorname{sen} z}{\cos z} dz =$$

$$= \ln |\operatorname{sen} z| - \ln |\cos z| + k =$$

$$= \ln \left| \frac{\operatorname{sen} z}{\cos z} \right| + k = \boxed{\ln \operatorname{tg} \frac{x}{2} + k}$$

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