$$\int \frac{3x-4}{x^2+x+4} =$$

INT-S1-027

$$D(x^2+x+4) = 2x+1$$

CERCO DI OTTENERE $2x+1$ AL NUMERATORE:

$$= \int \frac{\frac{3}{2}\left(2x-\frac{8}{3}\right)}{x^2+x+4}\,dx = \frac{3}{2}\int \frac{2x+1-1-\frac{8}{3}}{x^2+x+4}\,dx =$$

$$= \frac{3}{2}\int \frac{2x+1}{x^2+x+4}\,dx + \frac{3}{2}\int \frac{-1-\frac{8}{3}}{x^2+x+4}\,dx =$$

$$= \frac{3}{2}\ln(x^2+x+4) + \frac{3}{2}\int \frac{\frac{-3-8}{3}}{x^2+x+4}\,dx =$$

$$= \frac{3}{2}\ln(x^2+x+4) - \frac{11}{2}\int \frac{1}{\underbrace{x^2+x+4}_{\Delta = 1^2-16<0}}\,dx =$$

$$\rightarrow \quad x^2+x+4 = x^2+2\cdot\frac{1}{2}x+4 = x^2+2\cdot\frac{1}{2}x+\frac{1}{4}-\frac{1}{4}+4 =$$

$$= \left(x+\frac{1}{2}\right)^2 - \frac{1}{4}+4 = \left(x+\frac{1}{2}\right)^2+\frac{15}{4} =$$

$$= \frac{15}{4}\left[\frac{4}{15}\left(x+\frac{1}{2}\right)^2+1\right] = \frac{15}{4}\left[\left(\frac{2}{\sqrt{15}}\left(x+\frac{1}{2}\right)\right)^2+1\right]$$

SOSTITUZIONE: $\frac{2}{\sqrt{15}}\left(x+\frac{1}{2}\right) = z$

$$x+\frac{1}{2} = z\,\frac{\sqrt{15}}{2}$$

$$x = z\,\frac{\sqrt{15}}{2}-\frac{1}{2} \rightarrow x' = \frac{dx}{dz} = \frac{\sqrt{15}}{2}$$

$$dx = \frac{\sqrt{15}}{2}\,dz$$

$$= \frac{3}{2}\ln(x^2+x+4) - \frac{11}{2}\int \frac{1}{\frac{15}{4}(z^2+1)}\,\frac{\sqrt{15}}{2}\,dz =$$

$$= \frac{3}{2}\ln(x^2+x+4) - \frac{11}{\cancel{2}}\,\frac{\cancel{4}}{15}\,\frac{\sqrt{15}}{\cancel{2}}\int \frac{1}{z^2+1}\,dz = \boxed{\frac{3}{2}\ln(x^2+x+4) - \frac{11}{\sqrt{15}}\operatorname{arctg}\left(\frac{2}{\sqrt{15}}x+\frac{1}{\sqrt{15}}\right)}$$