

$$\int \frac{1}{1+e^x} dx =$$

INT-S1-021

SOSTITUZIONE: $1+e^x = z$

$$e^x = z - 1$$

$$x = \ln(z-1)$$

$$\frac{dx}{dz} = \frac{1}{z-1}$$

$$dx = \frac{1}{z-1} dz$$

$$= \int \frac{1}{z} \cdot \frac{1}{z-1} dz =$$

$$= \int \frac{A}{z} + \frac{B}{z-1} dz =$$

$$= \int \frac{A(z-1) + Bz}{z(z-1)} dz = \int \frac{z(A+B) - A}{z(z-1)} dz =$$

$$\begin{array}{ll} A+B=0 & B=1 \\ A=-1 & A=-1 \end{array}$$

$$= \int \frac{-1}{z} + \frac{1}{z-1} dz = -\ln|z| + \ln|z-1| + k =$$

$$= -\ln(1+e^x) + \ln e^x + k =$$

$$= \boxed{-\ln(1+e^x) + x + k}$$