$$\int \frac{1}{x^3(1+x^2)}\,dx =$$

INT-S1-005

$$= \int \frac{1+x^2-x^2}{x^3(1+x^2)}\,dx =$$

$$= \int \frac{\cancel{1+x^2}}{x^3\cancel{(1+x^2)}} - \frac{\cancel{x^2}}{x^{\cancel{3}}(1+x^2)}\,dx =$$

$$= \int \frac{1}{x^3} - \frac{1}{x(1+x^2)}\,dx =$$

$$= \int x^{-3}\,dx - \int \frac{1}{x(1+x^2)}\,dx =$$

$$= \frac{1}{-3+1}x^{-3+1} - \int \frac{1}{x(1+x^2)}\,dx =$$

$$= -\frac{1}{2}\cdot\frac{1}{x^2} - \int \frac{1}{x(1+x^2)}\,dx =$$

$$\downarrow$$

$$\frac{Ax+B}{1+x^2} + \frac{C}{x} = \frac{x(Ax+B)+C(1+x^2)}{x(1+x^2)} =$$

$$= \frac{Ax^2+Bx+C+Cx^2}{x(1+x^2)} =$$

$$= \frac{x^2(A+C)+Bx+C}{x(1+x^2)}$$ AFFINCHÈ SIA UGUALE A $\frac{1}{x(1+x^2)}$

DOVRÀ RISULTARE $\begin{cases} A+C=0 \\ B=0 \\ C=1 \end{cases} \rightarrow \begin{matrix} A=-1 \\ B=0 \\ C=1 \end{matrix}$

QUINDI: $\frac{1}{x(1+x^2)} = \frac{-x}{1+x^2} + \frac{1}{x}$

$$= -\frac{1}{2x^2} - \int \frac{-x}{1+x^2} + \frac{1}{x}\,dx = -\frac{1}{2x^2} + \int \frac{x}{1+x^2}\,dx - \int \frac{1}{x}\,dx =$$

$$= -\frac{1}{2x^2} + \frac{1}{2}\int \frac{2x}{1+x^2}\,dx - \int \frac{1}{x}\,dx = \boxed{-\frac{1}{2x^2} + \frac{1}{2}\ln(1+x^2) - \ln|x| + k}$$